Last week we discussed arithmetic means of arithmetic progressions in GMAT math problems. Today, let's see those concepts in action.

Question 1: If x is the sum of the even integers from 200 to 600 inclusive, and y is the number of even integers from 200 to 600 inclusive, what is the value of x + y?

(A) 200*400
(B) 201*400
(C) 200*402
(D) 201*401
(E) 400*401

Solution:

There are various ways of getting the answer here. We will use the concepts we learned last week.

The given sequence is 200, 202, 204, … 600

It is an arithmetic progression. What is the total number of terms here?

You can use one of two methods to get the number of terms here:

Method 1: Using Logic

In every 100 consecutive integers, there are 50 odd integers and 50 even integers. So we will get 50 even integers from each of 200 – 299, 300 – 399, 400 – 499 and 500 – 599 i.e. a total of 50*4 = 200 even integers. Also, since the sequence includes 600, number of even integers = 200 + 1 = 201

Method 2:

Recall that in our arithmetic progressions post, we saw that the last term of a sequence which has n terms will be first term + (n – 1)* common difference.

600 = 200 + (n – 1)*2

n = 201

Hence y = 201 (because y is the number of even integers from 200 to 600)

Let's go on now. What is the average of the sequence? Since it is an arithmetic progression with odd number of integers, the average must be the middle number i.e. 400.

Notice that since this arithmetic progressions looks like this:

(n – m), … (n – 6), (n – 4), (n – 2), n, (n + 2), (n + 4), (n + 6), … (n + m)

We can find the middle number i.e. the average by just averaging the first and the last terms.

[(n – m) + (n + m)]/2 = 2n/2 = n

Average = (200 + 600)/2 = 400

Sum of all terms in the sequence = x = Arithmetic Mean * Number of terms = 400*201

x + y = 400*201 + 201 = 401*201

Answer (D)

This question was simple. You could have found the sum using the formula n/2(2a + (n-1)d) that we saw in the AP post. But this method is more intuitive since if you don't want to, you don't have to use any formula here. Anyway, let's go on to our second question for today.

Question 2: The sum of n consecutive positive integers is 45. What is the value of n?

Statement I: n is even

Statement II: n < 9

Solution: First I will give the solution of this question and then discuss the logic used to solve it.

In how many ways can you write n consecutive integers such that their sum is 45? Let's see whether we can get such numbers for some values of n.

n = 1 -> Numbers: 45
n = 2 -> Numbers: 22 + 23 = 45
n = 3 -> Numbers: 14 + 15 + 16 = 45
n = 4 -> No such numbers
n = 5 -> Numbers: 7 + 8 + 9 + 10 + 11 = 45
n = 6 -> Numbers: 5 + 6 + 7 + 8 + 9 + 10 = 45

Let's stop right here.

Statement I: n must be even.

n could be 2 or 6. Statement I alone is not sufficient.

Statement II: n < 9
n can take many values less than 9 hence statement 2 alone is not sufficient.

Both statements together: Since n can take values 2 or 6 which are even and less than 9, both statements together are not sufficient.

Answer (E)

Now, the interesting thing is how do we get these numbers for different values of n. How do we know the values that n can take? It's pretty easy really. Follow my thought here.

Of course, n can be 1. In that case we have only one number i.e. 45.

n can be 2. Why? When we divide 45 by 2, we get 22.5. Since 2*22.5 is 45, we have to find 2 consecutive integers such that their arithmetic mean is 22.5. The integers are obviously 22 and 23.

n can be 3. When we divide 45 by 3, we get 15. So we need 3 consecutive integers such that their mean is 15. They are 14, 15, 16.

When we divide 45 by 4, we get 11.25. Do we have 4 consecutive integers such that their mean is 11.25? No, because mean of even number of consecutive integers is always of the form x.5.

n can be 5. When we divide 45 by 5, we get 9 so we need 5 consecutive integers such that their mean is 9. They must be 7, 8, 9, 10, 11.

n can be 6. When we divide 45 by 6, we get 7.5. We need 6 consecutive integers such that their mean is 7.5. The integers are 5, 6, 7, 8, 9, 10

Obviously, we just need to focus on getting 2 even values of n which are less than 9. So we check for 2, 4 and 6 and we immediately know that the answer is (E). We don’t have to do this process for all numbers less than 9 and we don’t have to do it for odd values of n.

We will move on to median next week. Till then, keep practicing!